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Numerical simulation of transients in single-reed woodwind instruments

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Abstract
In this work, we discuss a mathematical model of the sound production in single-reed woodwind instruments focusing on player-instrument interactions. The model consists of a coupled system of an ordinary and a partial differential equation. The motion of the reed is described by a nonlinear harmonic oscillator, where the nonlinearity stems from the collision of the reed with the mouthpiece lay and the coupling between air flow and pressure. The dynamics of air pressure and air velocity inside the instrument are described by a boundary value problem for the wave equation. We introduce a method to model the interaction between the player's tongue and the reed, which leads to a further nonlinearity in the reed equation. Moreover, we study strategies to estimate the model parameters relevant for the production of realistic tones, as well as for realistic transients in between tones. Finally, using the proposed model, recordings of real players are resynthesised.
Keywords: Single-reed, Transients, Articulation

1 INTRODUCTION

In order to synthesise sounds of musical instruments, numerical simulations of the physical processes that generate the respective sounds have been developed in the past decades [1, 13, 14]. Besides obtaining realistic sound waves, such an approach also allows to study the sound excitations mechanism, in terms of the underlying physics. In this framework, the player-instrument interaction can be seen as a further component that is included in the physical model. The scope of this paper is to incorporate player control in physical models of single-reed woodwind instruments. One main part of the player-instrument interaction takes place in the player's mouth by either changing the embouchure or by directly altering the vibration of the reed with the tongue [6, 11]. In this work, we describe a method to include the tongue-reed interaction into the physics-based synthesis of sounds of single-reed woodwind instruments. In an attempt to resynthesise recordings from real players, an inverse modelling approach is applied.

2 PHYSICAL MODEL OF A SINGLE-REED WOODWIND INSTRUMENT

2.1 Vibrations of a reed

We identify the whole reed with its tip, and model the vibration of the reed as a harmonic oscillator for the displacement $y(t)$ of the reed tip. See Figure 1 for a sketch of the system. During motion, the reed collides with the mouthpiece lay. The collision force is modelled following the Hunt-Crossley impact model [7]. Denoting the reed area by S_r and assuming an effective reed mass m, damping γ and stiffness k, the pressure difference $p_\Delta = p_\mathrm{b} - p_\mathrm{in}$ across the reed drives the reed tip $y(t)$ according to

$$m\frac{\mathrm{d}^2y}{\mathrm{d}t^2} + m\gamma\frac{\mathrm{d}y}{\mathrm{d}t} + ky + k_\mathrm{lay}\lfloor y - y_\mathrm{lay}\rfloor^{\alpha_\mathrm{lay}}\left(1 + r_\mathrm{lay}\frac{\mathrm{d}y}{\mathrm{d}t}\right) = S_r p_\Delta. \tag{1}$$

The parameters k_lay, r_lay and α_lay denote the contact stiffness, contact damping and the collision exponent of the Hunt-Crossley model for the collision between the reed and the mouthpiece lay. Furthermore, $\lfloor y\rfloor = yH(y)$,

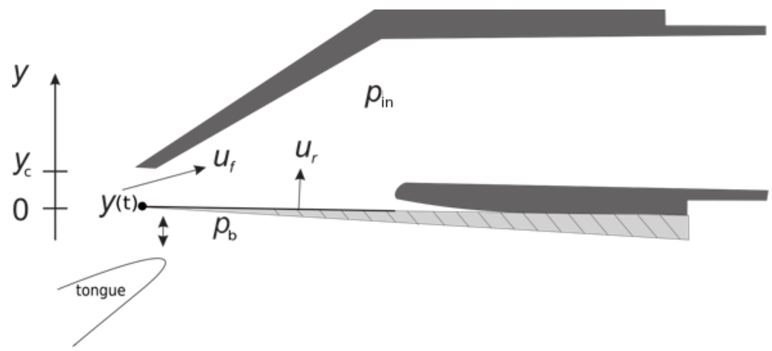

Figure 1. Single-reed – mouthpiece system. $y(t)$ denotes the reed displacement, y_c the displacement when the reed closes the mouthpiece, p_b the blowing pressure, p_{in} the pressure at the beginning of the tube, u_f the Bernoulli flow and u_r the reed induced flow.

where H denotes the Heaviside step function. The collision force becomes active whenever $y > y_{lay}$. Two types of air flow into the mouthpiece are considered. First, the motion of the reed induces the air flow $u_r = S_r \frac{dy}{dt}$. The pressure difference p_Δ introduces air flow into the mouthpiece according to Bernoulli's principle, $u_f = \sigma\lambda h\sqrt{\frac{2|p_\Delta|}{\rho}}$. Here, $\sigma = \operatorname{sign} p_\Delta$, λ denotes the reed width and $h = \lfloor y_c - y \rfloor$ denotes the reed opening, hence the product λh is the cross-sectional area of the opening. The total air flow into the mouthpiece is therefore given by

$$u_{in} = u_r + u_f = S_r \frac{dy}{dt} + \sigma\lambda h\sqrt{\frac{2\,|p_\Delta|}{\rho}}, \tag{2}$$

where ρ denotes air density.

2.2 Dynamics of pressure and particle velocity inside a tube

The corpus of the single-reed woodwind instrument is assumed to be a tubular duct of length L and cross-sectional area $S(x),\ 0 \leq x \leq L$. The spatio-temporal behaviour of the pressure p and particle velocity v inside the tube is modelled by [2, 12]

$$\frac{\partial p(x,t)}{\partial x} + \rho\frac{\partial v(x,t)}{\partial t} + z_v * v(x,t) = 0, \tag{3a}$$

$$\frac{\partial (S(x)v(x,t))}{\partial x} + \frac{S(x)}{\rho c^2}\frac{\partial p(x,t)}{\partial t} + S(x)y_\theta * p(x,t) = 0, \tag{3b}$$

where c is the speed of sound. The convolutions $z_v * v$ and $y_\theta * p$ are performed with respect to the time variable t and model viscous and thermal losses, respectively, inside the tube. The impedance z_v and the addmitance y_θ are defined in the frequency-domain, see [8]. To complete the model, boundary conditions have to be imposed. At the radiating end $x = L$, a stipulated radiation impedance z_r leads to the boundary condition

$$p(L,t) = S(L)z_r * v(L,t). \tag{4}$$

Specific forms of z_r can be found in [1, Section 9.1.3].

2.3 Reed-tube coupling

At the excitation end $x = 0$, the tube is coupled to the motion of the reed by the assumption that the air flow is continous, hence the air flow at the beginning of the tube is equal to the airflow coming into the tube,

$$S(0)v(0,t) = u_{\text{in}}. \tag{5}$$

Furthermore the pressure difference p_Δ is related to $p(0,t)$ by $p_{\text{in}} = p(0,t)$, i.e.

$$p_\Delta = p_{\text{b}} - p(0,t), \tag{6}$$

where p_{b} is the blowing pressure.

3 PHYSICAL MODEL OF THE TONGUE - REED INTERACTION

We aim at modelling the interaction of a player's tongue with the reed. For certain articulation techniques, the player stops and releases the vibrating reed with the tongue [11]. For this work, we assume that the tongue motion is perpendicular to the reed surface, as indicated in Figure 1. In order to model this phenomenon, we include an extra collision term in the equation of the reed movement according to the Hunt-Crossley impact model, such that the reed equation (1) now reads as

$$m\frac{\mathrm{d}^2 y}{\mathrm{d}t^2} + m\gamma\frac{\mathrm{d}y}{\mathrm{d}t} + ky + k_{\text{lay}}\lfloor y - y_{\text{lay}}\rfloor^{\alpha_{\text{lay}}}\left(1 + r_{\text{lay}}\frac{\mathrm{d}y}{\mathrm{d}t}\right) \tag{7a}$$

$$- k_{\text{tg}}\lfloor y_{\text{tg}} - y\rfloor^{\alpha_{\text{tg}}}\left(1 + r_{\text{tg}}\frac{\mathrm{d}(y_{\text{tg}} - y)}{\mathrm{d}t}\right) = S_r p_\Delta. \tag{7b}$$

Again, k_{tg}, r_{tg} denote contact stiffness and damping, and α_{tg} denotes the collision exponent. As opposed to the mouthpiece lay, the tongue is moving itself, hence we also have to take the velocity of the tongue $\frac{dy_{\text{tg}}}{dt}$ into account.

For synthesising the sound of an actual single-reed woodwind instrument, the model given by equations (3), (4), (5), (6) and (7) is discretised using a finite difference method that satisfies an energy balance, see for example [3, 5]. The special treatment of the visco-thermal losses related convolutions in Equation (3) is discussed in [12]. The finite difference method gives approximations p_m^n, v_m^n and y^n of the pressure, the particle velocity and the reed displacement, respectively at discrete timesteps $t^n = n\Delta_t$, $n = 0,1,2\ldots$ and position $x = m\Delta_x$, $m = 0,1,\ldots,M$, with $L = M\Delta_x$. A more detailed description together with a discussion of a finite difference method for the numerical implementation can be found in [4].

4 EXPERIMENTAL SETUP

In order to obtain reference measurements regarding the clarinet playing technique, a professional player from the Vienna Symphonic Orchestra was invited to perform musical excerpts using an equipped clarinet. The experimental setup consists of measuring the acoustic pressure in the mouthpiece of the clarinet and in the mouth of the player, together with the reed oscillation. For the mouthpiece pressure, a pressure transducer (Endevco 8507C-2) is inserted into the mouthpiece through a drill-hole at 7.5 cm from the mouthpiece tip. An identical pressure transducer is placed at the side of the mouthpiece and it remains inside the player's mouth during playing. In addition to that, we obtain information regarding the reed bending by attaching a strain-gauge on the surface of the reed. This signal allows to identify the tongue-reed contact, which is detected when the reed stops vibrating and closes against the lay of the mouthpiece. During the steady part of the signal, i.e. where there is no interaction between the player's tongue and the reed, the measured bending signal can be converted to reed displacement, as described in [10].

5 PARAMETER ESTIMATION

Our aim is to resynthesise an excerpt from the Clarinet Concerto No. 2, by Carl Maria von Weber (see Figure 2), based on the signals recorded with real players.

Figure 2. Excerpt from the Clarinet Concerto No. 2, by Carl Maria von Weber. The analysed part is highlighted.

To do that, we estimate the model parameters by comparing recorded mouthpiece pressure and reed displacement (obtained from the recorded reed bending) with their synthesised counterparts. In this work, we apply the Levenberg-Marquardt method [9, Chapter 10], which is an iterative optimisation technique that interpolates between Gauß-Newton's method and the gradient descent method.

In a first step, we obtain the effective tube length L for each single note. Since we are not modelling toneholes, the effective tube length might be smaller than the actual tube length. This depends on the clarinet register, with the discrepancy being insignificant in the lower register. A steady part of the note is first considered and the Levenberg-Marquardt method is applied to minimise the objective function

$$f_{\text{obj}} = \sqrt{\left(\frac{\|y(t)-\hat{y}\|}{\|y(t)\|}\right) + \left(\frac{\|p(0.02,t)-\hat{p}\|}{\|p(0.02,t)\|}\right)}, \tag{8}$$

where $\hat{y}$ is the reed signal computed from the bending measurement, $\hat{p}$ is the measured mouthpiece pressure, y is the modelled reed tip displacement and p here denotes the modelled pressure, obtained close to the excitation end. For the minimisation, the synthesised pressure is evaluated at $x = 0.02$m, i.e. two cm from the effective beginning of the tube. The symbol $\|\cdot\|$ denotes the ℓ^2-norm with respect to the discrete time samples under consideration.

Other reed parameters such as mass m, damping γ and stiffness k, or the reed-lay collision parameters k_{lay}, r_{lay} and α_{lay} do not have a direct physical counterpart, hence they are of effective nature as well. One reason for that is that the reed is modelled by a simple harmonic oscillator that aims to capture the effect of the player's embouchure. Therefore, having found the effective length L, we perform a second run on the same steady part of the signal in order to estimate the remaining reed-related parameters.

We model the motion of the tongue as a piecewise linear function, i.e. we assume that the tongue moves with constant velocity. During steady tones, the tongue rests at a resting position, where it does not interact with the reed. For each transient, the tongue starts moving upwards towards the reed at a time t_1 linearly from its resting position to its maximal value $y_{\text{tg,max}}$, which it attains at time t_2, and leaves at time t_3 in order to fall back to its resting position at time t_4. See Figure 3 for an illustration. The resting position is kept fixed. We obtain the parameters related to the motion of the tongue t_1, t_2, t_3, t_4 and $y_{\text{tg,max}}$ and the parameters related to the collision of the tongue with the reed, k_{tg}, r_{tg} and α_{tg} by applying the Levenberg-Marquardt method on the transient parts of the signal.

Since the reed displacement obtained from the bending measurement is not reliable during reed - tongue contact, we modify our objective function in a way that it ignores the reed signal during contact, i.e.

$$f_{\text{obj}} = \sqrt{\left(\frac{\|H(y-y_{\text{tg}})(y(t)-\hat{y})\|}{\|H(y-y_{\text{tg}})y(t)\|}\right) + \left(\frac{\|p(0.02,t)-\hat{p}\|}{\|p(0.02,t)\|}\right)}. \tag{9}$$

Behold that $H(y-y_{\text{tg}})$ is 0 whenever $y < y_{\text{tg}}$, i.e. when the tongue is in contact with the reed, and 1 otherwise.

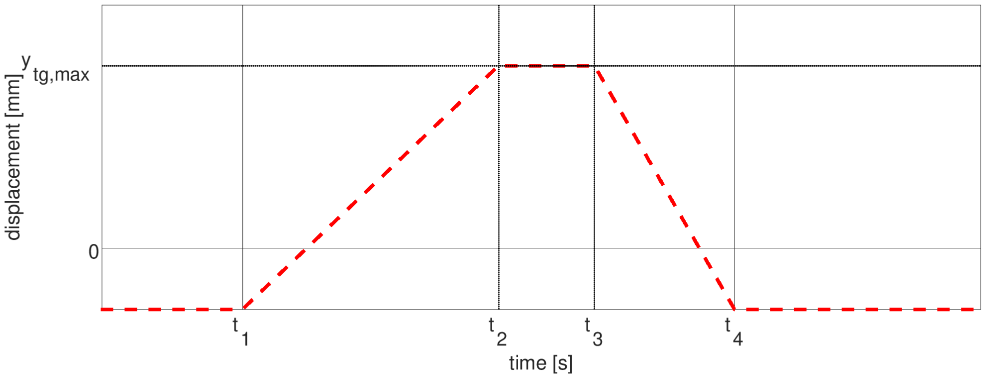

Figure 3. Illustration of the modelled motion of the tongue.

Figure 4 shows the estimated tongue movement together with the synthesised reed displacement and the synthesised mouthpiece pressure corresponding to the highlighted phrase of the excerpt in Figure 2. The synthesised signals are qualitatively similar to the reference signals in Figure 5.

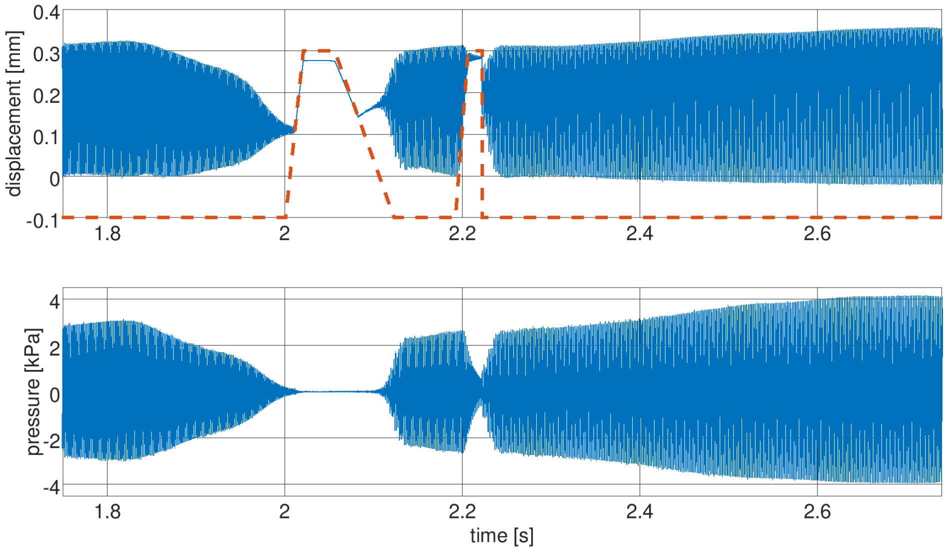

Figure 4. Top: Synthesised reed displacement (solid blue) and tongue displacement (dashed red) in mm, for the note transitions highlighted in Figure 2. Bottom: Synthesised mouthpiece pressure.

6 CONCLUSION

This work discusses a physical model for the tongue-reed interaction while simulating the sound generation of a single-reed woodwind instrument. This is established by including a further collision term into the equations that describe the oscillation of the reed in the case of tongue interference. Using the recorded pressure and reed displacement signals obtained from measurements with real players, an optimisation procedure finds pa-

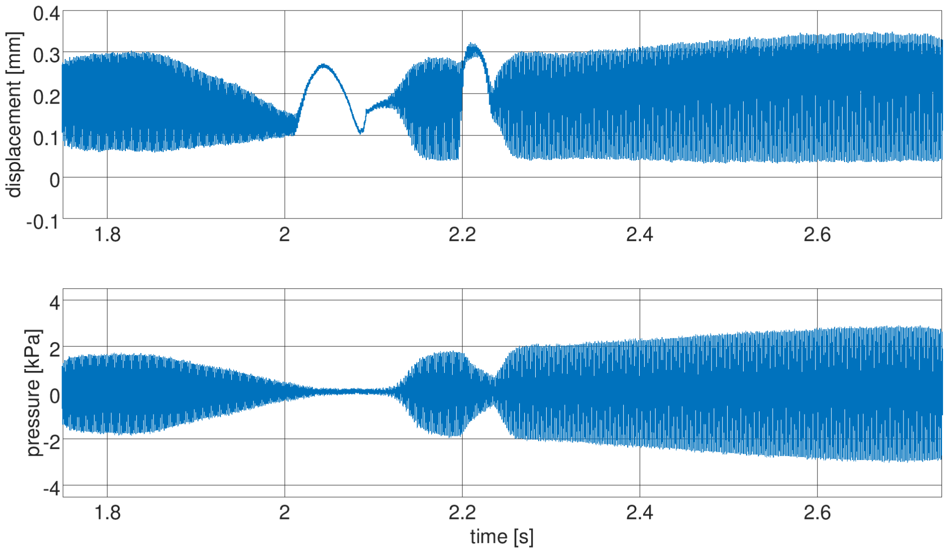

Figure 5. Signals obtained from measurements with a professional clarinettist. Top: Reference reed displacement. Bottom: Reference mouthpiece pressure.

rameters for which the physical model synthesises pressure and reed signals that are qualitatively similar to the recordings.

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